The tale of two sellers: Risk-shifting through house listing prices among sellers with high and low loan-to-value ratios

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In the residential real estate market, sellers with different unobservable characteristics experience different outcomes with regards to their time to sale. One such characteristic is the seller's loan-to-value ratio. Empirical evidence shows that the higher the seller's LTV ratio is, the longer it takes them to sell their property. However, there is a tradeoff between the time to sale and the final sale price that the seller receives for their house. This relationship can be interpreted as a risk-shifting problem where the homeowner increases the risk associated with the house sale by increasing the listing price, which results in a longer time to sale. As a result, the homeowner enjoys a larger upside (higher proceeds from the house sale) in case of success, and the bank bears a higher downside in case the house is not sold. In this paper, I develop a model following Jensen and Meckling's (1976) model of leverage-induced risk shifting that explains why sellers with higher LTV ratios list their properties at higher prices.[1]

Keywords: real estate, loan to value ratio, time to sale, default rate, residual income, sellers, housing market

[1]This is a preliminary draft of the paper and is subject to reviews and changes. I thank Anjan Thakor and my classmates who participated in the Information Economics and Corporate Finance Theory course at Olin Business School (Spring 2020) for helpful comments and suggestions.

1 Introduction

"Time is money." This popular saying has existed for a very long time, and we are taught to believe that the faster our actions are and the faster some event happens, the lower our losses will be. We are often taught to not "waste" our time contemplating and exploring options. But does it always work that way and should it be the goal for everyone? A stylized example from the residential real estate market shows that the truth is somewhere in the middle. When we look at different sellers listing their houses on the market, we find that they may have different objectives. These objectives depend on certain unobservable characteristics. While for some sellers selling faster is their major objective, for others waiting for a higher selling price is more important, even if it leads to a longer time to sale. Moreover, sellers want to ensure that they are not the only ones bearing the risk that they incur by putting their house on the market. One key feature that distinguishes different sellers is their loan-to-value ratio. Empirical evidence shows that sellers with a higher loan-to-value ration tend to set a higher listing price and thus have a longer time to sale than the sellers with lower loan-to-value ratios. Why is there such heterogeneity among sellers based on the loan-to-value ratio?

In this paper, I solve this problem from the perspective of leverage-induced risk-shifting for sellers with different loan-to-value (LTV) ratios, following Jensen and Meckling (1976) [8]. I view selling the house as a project, where the seller holds equity in their property and the creditor (the bank) is a debtholder. I also incorporate empirical motivation from Genesove and Mayer (1997, 2001) to set up the LTV ratio of the seller. [5] [6]

The first key finding is that sellers with high LTV ratios indeed set and receive higher prices for their listings, as they are willing to engage in risky pricing. From the bank's perspective, claims are maximized with such homeowners. However, banks also face higher potential losses from lending to homeowners who contribute very little equity to the house value. The second key finding is that higher interest rates set by the banks lead to homeowners putting more equity towards their home purchase. Contributing more equity to the house value leads to lower interest rates from the bank. If the time to maturity is short, the interest rate becomes very low even with a relatively small equity contribution. Shorter times to maturity signal faster repayment, and thus less risk for the bank.

My contribution to the existing literature is that I attempt to explain the phenomenon of higher listing prices by more indebted house owners through a model that focuses on risk-shifting done by sellers. I also introduce the use of call option as a way to describe how the listing price and the equity contribution of the homeowner to the house value affect the bank's decision to give the homeowner a certain interest rate at the time of mortgage origination. My work is the most similar to the paper by Head et al. (2021), who study the effects of household indebtedness on house selling decisions. They develop a dynamic model with search in the housing market and defaultable long-term mortgages. The main finding is that in equilibrium, both sellers' asking prices and time-to-sell increase with the relative size of the owners' outstanding debt. For sellers in financial distress, this results in higher risk of default. They calibrate the model to the U.S. economy and find positive correlations of asking prices with LTV ratios across sellers. They also find positive correlations of house prices and time-to-sell with origination LTVs over time. [7]

The difference of my model from the model proposed by Head et al. (2021) is that I make an assumption that sellers view house sale as a capital project, with them owning equity in their own home and the bank being a debtholder. Head et al. instead focus on the likelihood of default being the main driver for the chosen listing prices. Moreover, my model allows for the possibility of homes being owned by those who don't occupy them, which is not accounted for in the model by Head et al. This results in allowing for sellers with high LTV ratios who face a relatively low chance of default and are risk-neutral or even risk-seeking.
While the main findings of higher LTV ratios being associated with higher sale prices are similar to my paper, Head et al. don't explore in detail any interactions of interest rates with the amount of equity put towards the house or with the times to maturity. Instead, the interest rates are considered only as a part of the "lending standards" at origination and only affect mortgage outcomes via origination pricing.

2 Motivation and literature review

2.1 Heterogeneity in listing prices

In a residential real estate market, different types of sellers experience different outcomes when it comes to time-to-sale. Many homeowners are interested in selling their house in a minimal amount of time possible for a maximum price possible due to time constraints that they face. How fast a house is being sold depends on the initial list price, which is caused in part by unobserved characteristics of sellers [1]. These characteristics can be combined into two groups of positive and negative characteristics, which will define "good" and "bad" sellers. One such characteristic that has been discussed in empirical literature is loan-to-value ratio of the seller: "good" sellers are those with low LTV ratios, and "bad" sellers are those with high LTV ratios. One regularity is that "bad" sellers tend to list their houses at a higher price than "good" sellers. Another regularity is that "bad" sellers take longer to actually sell the property. Genesove and Mayer (1997) show that sellers with a higher loan-to-value ratio set a higher list price and have a longer time to sale than those with a lower loan-to-value ratio. They find that within any given amount of time, a property that was 100% financed by a mortgage (loan-to-value ratio of 1) is 16% less likely to sell than a property with no mortgage. [5]

However, selling price reductions are shown to be very beneficial to the sellers. De Wit and van der Klaauw (2013) in a survey of Dutch administrative data find that a list-price reduction raises the selling rate (probability of a house being sold) by 83%, effectively reducing the time on the market. There seems to be a major literature gap in understanding why sellers with higher LTV ratios set higher initial listing prices higher than those with lower LTV ratios, even though they know it may cost them additional time on the market. [4]

2.2 Literature review

The effect of the listing price on the property sale outcomes has been discussed by many scholars. Albrecht et al. (2016) find that houses can be sold above, below, or at the asking price, depending on unobserved characteristics of sellers. They also find that the asking price can signal seller's type. [1] Deng et al. (2012) find that less-informed sellers set a higher list price than better-informed sellers and that their properties stay longer on the market. [3]

There is also some research dedicated to sale price reductions. De Wit and van der Klaauw (2013) find that listing price reductions raise the selling rate by 83%. Moreover, the longer the property has been staying on the market, the stronger the effect. The strongest effect is observed for properties that were repriced after 182 days ($\approx$ 6 months) on the market. [4] Sale price reductions are also more likely to be done by certain types of sellers; Deng et al. (2012) show that less-informed sellers are more prone to reducing their listing price. [3]

A few papers discuss how sellers with different loan-to-value ratios price their properties. Genesove and Mayer (1997) use LINK and Banker and Tradesman data from the Boston condominium market (1990-1992) to determine the relationship between an owner's home equity and corresponding loan-to-value ratio (which is defined by the amount of mortgage debt relative to the property value) and the price that they ask for their property. The study included both owner-occupants and property investors (sellers who own condominiums and rent them out). They use hazard rate of sale for each individual property and relative hazard rates to find which properties will be more likely to sell. They find that within any given period of time, properties that were fully financed by a mortgage are 16% less likely to sell than properties that were financed without any incurred debt. When the loan-to-value ratio is even slightly above 1, the probability of the house being sold decreases further: properties with LTV ratios of 1.1 were 24% less likely to sell than no-mortgage properties. In other words, it takes longer for high LTV properties to sell than for properties with minimal to no debt. [5]

However, Genesove and Mayer also find that sellers with high loan-to-value ratios (regardless of whether they live in their home or rent it out) set higher listing prices than sellers with low loan-to-value ratios. Moreover, sellers with high LTV ratios tend to have a higher reservation price and also tend to get a higher final sales price than sellers with low LTV ratios. A few explanations are offered to this regularity, though not all of them are fully satisfying. Investors tend to set higher listing prices, because they (1) can afford to do that, since rent and tax depreciation are enough to cover mortgage payments, and (2) they have a higher reservation price that they would be willing to accept in order to avoid default on the property, so they are holding off the sale until they get an acceptable offer. So it's actually in their best interest to stay on the market longer rather than sell the property faster. When discussing owner-occupants, Genesove and Mayer suggest that they tend to sell their homes faster than investors – but are more willing to accept any offer that they receive, even if it's at a large discount from their asking price. Owner-occupants generally have higher costs of default – which include a high likelihood of not being able to purchase another home for 7-10 years and possible seizure of other personal assets. Moreover, if they wait for too long to accept an offer, having to rent a home and looking for a new home are additional costs that sellers will have to incur. [5] A later paper by Genesove and Mayer (2001) attributes the fact that sellers with high LTV ratios set higher listing prices to loss aversion. [6] This result is confirmed by Head et al. (2021) in their dynamic model of housing and mortgage markets. [7]

My main theoretical reference is the managerial behavior, agency costs and ownership structure model by Jensen and Meckling (1976), which establishes among other things a notion of risk-shifting by equityholders. In my setting, homeowners effectively become equityholders; they grow their stake as they pay off their mortgage and can even obtain loans against their home equity. Trivially, the bank is a debtholder. [8] I also use Black-Scholes (1973) model for option valuation/pricing to solve homeowner and bank problems. [2]

3 Model

3.1 Key variables and assumptions

Assume there are two types of house sellers: (1) sellers with low LTV (type L), and (2) sellers with high LTV (type H). All sellers are fully aware of the property market price, denoted further as P_m. This is the value of the house according to the most recent market appraisal. The net proceeds from the house sale N are used to repay the bank, since the bank gives the seller a call option at t=0. All sellers' preferences are homothetic. I also consider 2 states γ of the economy: good (denoted as G) and bad (denoted as B). Finally, assume that there are no costs associated with listing the house on the market and that there are no property tax hikes associated with market value increases.

Let P_0 be the initial risky listing price of the seller:

$$P_{0,HLTV} > P_{0,LLTV};$$

$$P_{0,G} > P_{0,B}.$$

This is the price that the seller chooses to list their house at; it may be a price that is close to the market price (see notation for Pm) or it may be very different from P_m. Sellers with high LTV ratios list their houses at a higher price than those with low LTV ratios. In the good state of the economy, sellers tend to list their houses at a higher price than in the bad state of the economy, since there should be a higher demand for houses when the economy is doing well.

P_{min} is the reservation price of the seller, that is, the minimum price that the seller is willing to accept when selling their house:

$$P_{min,HLTV} \geq P_{min,LLTV} > 0;$$
$$P_0 \geq P_{min}.$$

Since sellers with lower LTV ratios sell their houses faster and at a lower final sale price than those with high LTV ratios, it should be that their reservation price is lower (even if not by a lot). They don't have to worry about defaulting on their loan to the same extent as sellers with high loan-to-value ratios and have positive home equity, so such sellers need not sell their house at a substantial premium. For all sellers, the listing price should be at least as large as their reservation price.

Pm is the estimated riskless market price of the property:

$$P_m \geq 0,$$
$$P_{0,HLTV} \geq P_{0,LLTV} \geq P_m.$$

The initial listing price for all sellers is at least as large as the market price. If the sellers list their house at this price, they are guaranteed a sale with $p_{sale} = 1$. Their net proceeds N will be 0. If sellers choose to list their house at a P_0 that differs from the market price, they expose themselves to a risk of selling at a price below P_m and thus not being able to collect a positive profit on the house sale.

P_{sell} is final sale price of the property given to the seller by a market participant buying the house (the buyer):

$$P_{sell} = \rho P_{sell,G} + (1-\rho)P_{sell,B}, P_{sell,G} > P_{sell,B}.$$

where ρ is the probability that $\gamma = G$, $P_{sell,G}$ is the selling price in case the economy is good, and $P_{sell,B}$ is the selling price in case the economy is bad.

$$P_{sell,HLTV} > P_{sell,LLTV} > 0 \text{ in any state of the economy;}$$
$$P_{sell} \geq P_{min} > 0 \text{ in any state of the economy;}$$
$$P_m \leq P_{sell} \leq P_0 \text{ if } \gamma = G,$$
$$P_{sell} \leq P_0 \leq P_m \text{ if } \gamma = B.$$

The final sale price of the property has to be at least as good as the reservation price. Given empirical evidence from Genesove and Mayer (1997, 2001), I set the first inequality: investors with high LTV ratio sell properties at a higher final price than sellers with a low LTV ratio. The final sale price cannot be larger than the listing price.

L is the LTV ratio of the seller:

$$0 < L < 2; 0 \leq L_L \leq 0.5; 0.5 < L_H \leq 2.$$

I assume that the cutoff between low and high loan-to-value ratio is at 50%. This is a voluntary number; the empirical papers that I have reviewed tend to compare no-debt sellers to all-debt sellers. Though Genesove and Mayer (1997) include LTV ratios that are above 1, I will follow the traditional structure of L+E = 1.

$D(L, V_h)$ is the mortgage value function:

$$D_{t=0} = L * V_h = V_h * (1 - E),$$

where L is the loan-to-value ratio of the seller and V_h is the house value at t=0:

$$0 \leq D_{LLTV} < D_{HLTV}.$$

This is the value of the mortgage at t=0, when the individual first acquires the house. Since the interest rate r is fixed, the principal stays the same throughout t=T. The amount that the seller is expected to repay the bank at t=T is equal to

$$D_{t=T} = D_{t=0}(1 + r)^T,$$

where r is the interest rate defined by a functional form $r(V_h, E)$. Further, we can break the mortgage value to

$$D_{t=T} = D_{t=0} + I_T = V_h * (1 - E) + I_T,$$

where I_T is the interest on the loan received by the bank at the time of house sale.

$C(P_sell, D)$ is the European call option given to the homeowner by the bank:

$$C^+ = P_{sell} - D \geq 0 \text{ if } P_{sell} > D,$$
$$C^- = 0 \text{ if } P_{sell} < D,$$
$$C = \theta C^+ + (1 - \theta)C^-,$$

where $0 \leq \theta \leq 1$ is the probability of successful sale.

The intuition behind this function comes from the fact that the bank will only get paid if the sale price is greater than the value of debt. If the seller fails to get a sale price that covers the principal and the interest on the mortgage at time T, his payoff will be 0. Also, if the seller has a 100% claim on their assets (in other words, the value of the home is equal to the value of the home equity), the bank will not get anything from them.

3.2 The timeline

At t=0: The homeowner purchases a house valued at V_h, puts up an equity value equal to $V_{h,E}$ and obtains a loan D at a fixed interest rate $r(V_h, E)$. The bank thus gets a claim on $V_h(1-E) \geq 0$.
At t=1: The homeowner lists a house for sale at P_0.
At t=S: The homeowner sells the house at P_{sell}.
At t=T: The homeowner has to repay $D(1+r)^T$ to the bank and collects N. P_{sell} earns no interest for the homeowner between t=S and t=T.

3.3 Model solution and propositions

3.3.1 Model with simple European call option

When the state of the economy is unknown, each seller is solving

$$max_{P_0} N = P_{sell} - D(L, V_h)(1+r)^T = P_{sell} - D_{t=T}(L, V_h) = C(P_{sell}, D) \geq 0 \tag{1}$$

$$s.t. max_{r,E} V_h - C(P_{sell}, D) = V_h - P_{sell} + D_{t=T}(L, V_h) \geq 0; \tag{2}$$

$$P_{0,HLTV} > P_{0,LLTV} \geq P_m; \tag{3}$$

$$P_{min,HLTV} > P_{min,LLTV} > 0; P_0 \geq P_{min}; \tag{4}$$

$$P_{sell,HLTV} > P_{sell,LLTV} > 0; \tag{5}$$

$$0 \leq D_{LLTV} < D_{HLTV}. \tag{6}$$

This leads me to the first proposition:
Proposition 1. In case of interest-free borrowing and in case $P_m = V_h$ (market value of the house not changing between t=0, t=S, and t=T), the net proceeds from the house sale will be maximized for sellers whose houses were fully financed by a mortgage, but the mortgage value does not exceed the house value V_h (an LTV ratio of 1). They can thus charge any price $P_0 > P_m$ that they deem appropriate. More specifically, this price will be $P_{0,HLTV} > P_m$. The bank's claim on the assets is also maximized.
Corollary 1. For unlevered sellers, it is optimal to sell their houses at P_m, since they bear the entire loss in case they charge a $P_{0,LLTV} > P_m$. Levered sellers with low LTV ratios should be charging a $P_{0,LLTV} < P_{0,HLTV}$, which converges to P_m as the equity contribution to V_h increases.
Assume r=0. We can rewrite the main equation as

$$max_{P_0} N = \theta[\rho P_{sell,G} + (1-\rho)P_{sell,B} - LP_m] \geq 0,$$

which yields

$$max_{P_0} N = \rho P_{0,G} + (1-\rho)P_{0,B} - LP_0 \geq 0.$$

Taking the first-order condition with respect to P_0, we get

$$\rho + (1-\rho) - L = 0,$$
$$L^* = 1,$$
$$E^* = 1 - L^* = 0,$$
$$P_0^* = D/L^* = D.$$

Since borrowing is interest-free in this case, it is optimal for the seller to be in an all-debt position. Because the seller does not own any equity in their home, they have the most appetite for risk and are willing to engage in risky pricing. That allows them to set the maximum possible listing price equal to the entire value of their mortgage, which will result in a higher sale price than for those who do have some equity in their home. Moreover, since the bank has the claim on the entire value of their home, all potential losses will be borne by them. The proof is provided in Appendix 1.

Proposition 1a. In case of the bank charging an interest rate on a mortgage and in case $P_m = V_h$, the optimal LTV ratio under which the net proceeds from the house sale will be maximized is defined by 1 over the discount factor at t=T. The ratio will decrease as the interest rate r increases. As the interest rate increases, owners will want to counteract by putting more equity against the loan when they first acquire a property at value V_h and get a mortgage $D_t = 0$.

In this case, the main equation becomes

$$max_{P_0} N = \theta[\rho P_{sell,G} + (1-\rho)P_{sell,B} - LP_m(1+r)^T] \geq 0,$$

which yields

$$max_{P_0} N = \rho P_{0,G} + (1-\rho)P_{0,B} - LP_0(1+r)^T \geq 0.$$

Taking the first-order condition with respect to P_0, we get

$$\rho + (1-\rho) - L(1+r)^T = 0,$$
$$L^* = 1/[(1+r)^T],$$
$$E^* = 1 - L^* = 1 - 1/[(1+r)^T],$$
$$r^* = \frac{1}{\sqrt[T]{L}} - 1.$$

As we can see, the proposition indeed holds. Another important outtake is that the interest rate is defined by a nonlinear function (which will be discussed further in proposition 2).

These findings are consistent with the empirical results stated earlier. Sellers with high LTV ratios indeed list and sell their houses at a higher price than those with low LTV ratios, indicating that the risk-taking associated with sellers' indebtedness pays off.

Proposition 2. The initial pricing of the bank loan is described by a nonlinear function. There is a level of equity E at which the price of the loan will start to decline.

Let us return to the problem solved by the bank:

$$max_{r,E} V_h - \theta[P_{sell} - D(1+r)^T] \geq 0$$
$$\text{s.t. } max_{P_0} N = C = P_{sell} - D(L, V_h)(1+r)^T \geq 0.$$

Taking the first-order condition with respect to E yields

$$FOC_E : P_m(1+r)^T = V_h(1+r)^T = 0,$$
$$E^* = 0.$$

Taking the first-order condition with respect to r yields

$$FOC_r : V_h(1-E)T(1+r)^{T-1} = 0.$$

Thus, the interest rate r is defined by the following equation derived from the FOC:

$$e^{V_h(1-E)T(1+r)^{T-1}} = 1.$$

For an all-debt homeowner, the FOC_r is

$$V_h T(1+r)^{T-1} = 0,$$

resulting in the interest rate r solving a nonlinear exponential equation

$$e^{V_h T(1+r)^{T-1}} = 1.$$

This leads to the following conclusions:

1. The interest rate r that the bank charges the homeowner solves a nonlinear exponential equation $e^{V_h(1-E)T(1+r)^{T-1}} = 1$. The interest rate r is a function of the house value V_h at the time of loan origination, the equity ratio E, and the time to maturity T. The bank will charge the maximum interest rate r to borrowers who put in no equity.

2. For a given house value and time to maturity, there is a level of equity E that minimizes the interest rate r at the time of loan origination t=0. The bank will maximize its proceeds when there is no equity by charging a higher interest rate on the loan.

Corollary 2a: The bank will give the homeowner the highest interest rate when $E \to 0$.

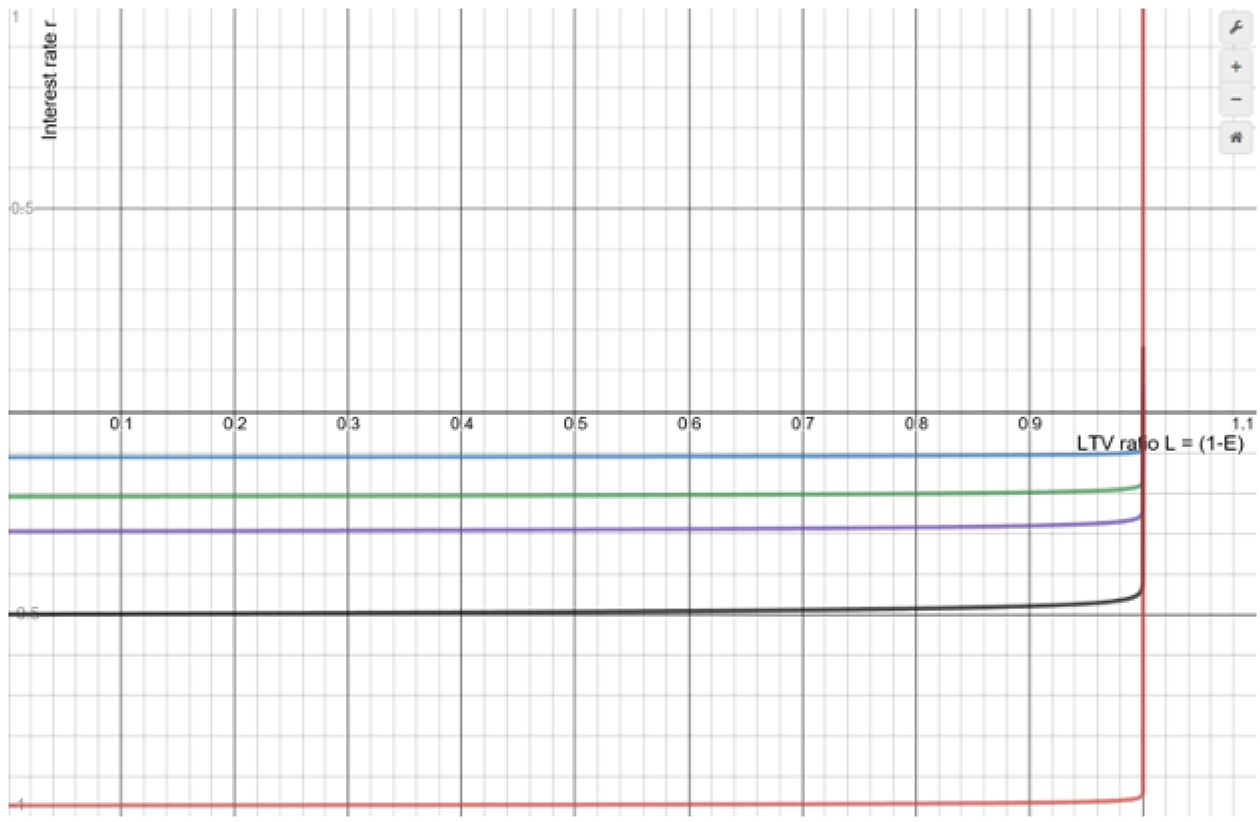

Figure 1: The relationship between the loan to value ratio and the interest rate r for T=360 months, 180 months, 120 months, 60 months, 12 months

Intuitively, there should be an optimal level of equity at which the bank is willing to give the homeowner the lowest interest rate on their mortgage. The bank knows that when the homeowner provides less equity when negotiating a mortgage, they will be more likely to engage in risky behavior in the future, since it will not affect their home equity as much as it would for someone with a larger equity stake. However, if the homeowner decides to take on a risky project (such as trying to list their home at a price above the market price), the downside will be absorbed by the debtholder. The bank would thus want to encourage the homeowner to provide more equity by giving them a low interest rate on their mortgage in order to divert them from risky behavior. Figure 1 demonstrates the relationship between the loan to value ratio and the interest rate r for different times to maturity. In all cases, there is an inflection point at $L \approx 1$ where the interest rate grows rapidly; the interest rate spikes when E=0.

3.3.2 Model solution with Black-Scholes European call option

Now, I solve the model using Black-Scholes option value formula. The value of the European call at t=T is defined for the homeowner and the bank as

$$C(P_{sell}, D, r, T, \sigma^2),$$
$$C = P_{sell}N(d_1) - N(d_2)De^{-rT},$$
$$d_1 = \frac{\ln\frac{1}{L} + (r + \frac{\sigma^2}{2}) * \Delta T}{\sigma\sqrt{\Delta T}},$$
$$d_2 = d_1 - \sigma\sqrt{\Delta T},$$

and N is the cumulative distribution function of the standard normal distribution.

The homeowner's problem becomes

$$max_{P_0} C = P_{sell}N(d_1) - N(d_2)De^{-rT} \geq 0,$$

$$s.t.max_{r,E}V_h – C(P_{sell}, D) = V_h - P_{sell}N(d_1) + N(d_2)De^{-rT} \geq 0, \tag{7}$$

$$P_{0,HLTV} > P_{0,LLTV} \geq P_m; \tag{8}$$

$$P_{min,HLTV} > P_{min,LLTV} > 0; P_0 \geq P_{min}; \tag{9}$$

$$P_{sell,HLTV} > P_{sell,LLTV} > 0; \tag{10}$$

$$0 \leq D_{LLTV} < D_{HLTV}. \tag{11}$$

The FOC with respect to P_0 is

$$N(d_1) - N(d_2)Le^{-rT} = 0,$$
$$L^* = e^{rT}\frac{N(d_1)}{N(d_2)},$$
$$E^* = 1 - e^{rT}\frac{N(d_1)}{N(d_2)},$$
$$r^* : e^{-rT}\frac{N(d_1)}{(1-E)N(d_2)} = 0.$$

The debtholder's problem is

$$max_{r,E}V_h – C(P_{sell}, D) = V_h - P_{sell}N(d_1) + N(d_2)De^{-rT} \geq 0,$$
$$s.t.max_{P_0}C = P_{sell}N(d_1) - N(d_2)De^{-rT} \geq 0, \tag{12}$$
$$(8), (9), (10), (11).$$

The FOC with respect to the interest rate r is

$$FOC_r : -N(d_2)(1 - E)e^{-rT} = 0.$$

The following formula gives a better description of the relationship between the equity to value ratio and the interest rate than the formula derived from the FOC based on the simple call option model. This brings yet another conclusion:

1. The interest rate r will decline as the equity ratio increases. Moreover, for shorter times to maturity, the decline will be faster than for longer times to maturity. In other words, the homeowner needs a lower equity contribution when the time to maturity is shorter to receive a lower interest rate with the bank.

Again, the relationship between the equity to value (or loan to value) ratio and the interest rate on the mortgage debt is nonlinear. But unlike the first case, where I explored such relationship in a simple call option setting, the formula derived from the Black-Scholes equation suggests a more gradual curve that connects the equity to value ratio and the interest rate. The interest rate also tends to decline faster for shorter terms. A potential empirical explanation could be that when a homeowner borrows with a promise to repay the debt very soon, the bank accepts such promise as a sign that the mortgage is not as risky as longer-term mortgages. As a result, the bank is willing to extend credit to such homeowners on more favorable terms. Another explanation could be that when potential homeowners look at the schedule of interest rates for different maturities, they purposefully make a choice towards shorter maturities: to get a lower rate, they will need to provide less equity than they would if they were to choose a longer loan term.

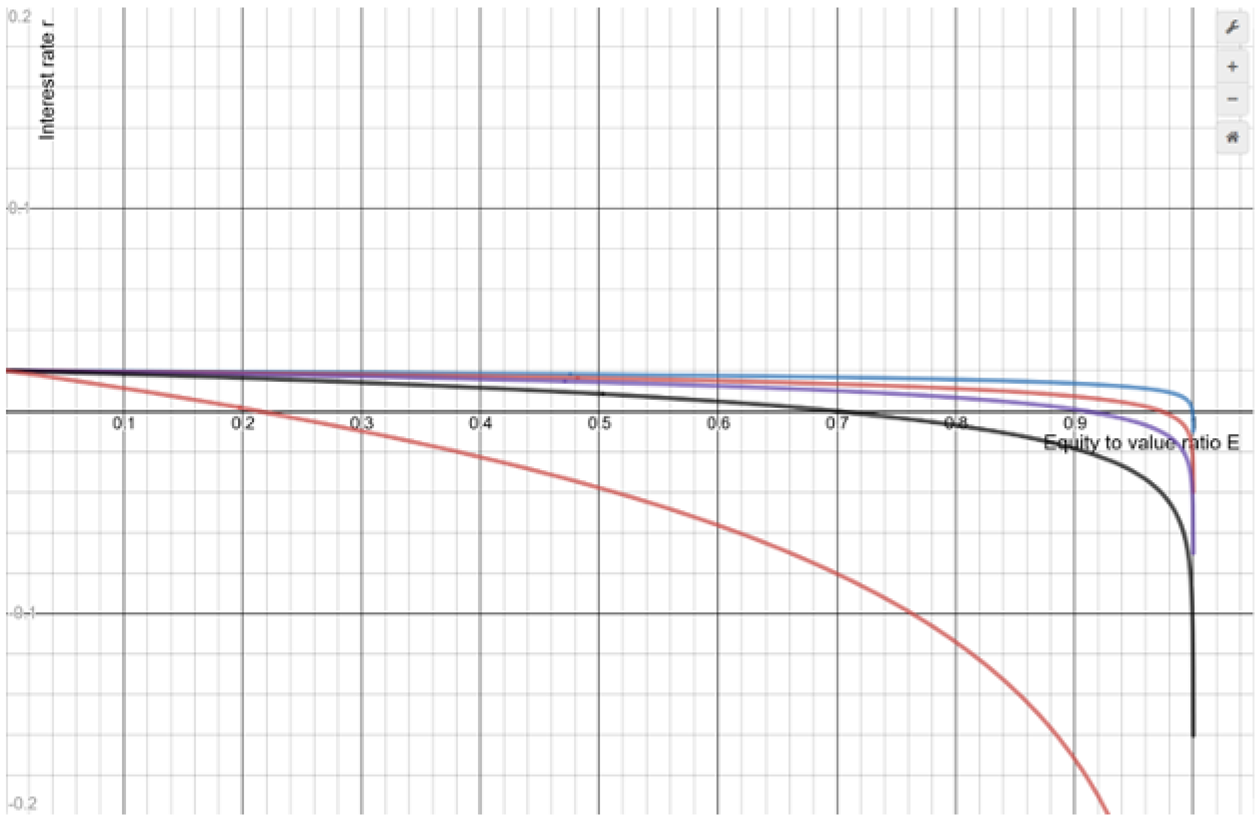

Figure 2: Figure 2: the relationship between the equity to value ratio and the interest rate r for T=360 months, 180 months, 120 months, 60 months, 12 months, $\sigma = .2$

The figure above demonstrates the statement. For instance, when the borrower promises to repay the mortgage in just a year (12 months), the interest rate turns to 0 when they make a mere 20% contribution to the house value. But those homeowners who take on 30-year (360-month) mortgages need a more than 4.5 times that contribution to get a lower interest rate. The main outtake is that putting equity towards the home value ultimately benefits the homeowner through a lower interest rate, while borrowing the entire amount through the bank benefits the bank, since they can collect a higher interest.

Proposition 3. The bank loss increases in mortgage value D and in initial listing price P_0. Moreover, the lower probability of successful sale is, the bigger loss will be.

3.3.3 Loss with simple European call option

I define the bank loss from the homeowner's failure as

$$F = [1 - \theta]C(P_{sell}, D),$$

where θ is the probability of successful sale.

Thus, the bank solves

$$min_L F = [1-\theta]C(P_{sell}, D) \geq 0,$$

$$s.t. max_{P_0} N = P_{sell} - D(L, V_h)(1+r)^T = P_{sell} - D_{t=T}(L, V_h) = C(P_{sell}, D) \geq 0; \quad (13)$$

$$max_{r,E} V_h - C(P_{sell}, D) = V_h - P_{sell} + D_{t=T}(L, V_h) \geq 0; \quad (14)$$

$$P_{0,HLTV} > P_{0,LLTV} \geq P_m; \quad (15)$$

$$P_{min,HLTV} > P_{min,LLTV} > 0; P_0 \geq P_{min}; \quad (16)$$

$$P_{sell,HLTV} > P_{sell,LLTV} > 0; \quad (17)$$

$$0 \leq D_{LLTV} < D_{HLTV}. \quad (18)$$

The equation can be restated as

$$[1-\theta](P_m - P_m L(1+r)^T) \geq 0.$$

Taking the first-order condition with respect to L results in

$$FOC_L : [\theta - 1]P_m(1+r)^T = 0,$$
$$L^* = 0,$$
$$E^* = 1 - L^* = 1.$$

The conclusions are as follows:

1. For the bank to not experience any losses, the borrower should have a loan-to-value ratio close to 0. Extremely risk-averse banks should not be extending credit to the borrowers.

2. If the bank decides to extend credit to the borrower, the probability of success should be equal to 1 for the bank to not experience any losses.

This statement is quite trivial: if the borrower is not dependent on the bank as a source of financing for their home purchase, the bank doesn't have to worry about any losses. But at the same time, the bank can't make a claim on the homeowner's assets if the homeowner is not borrowing anything. The proofs are provided in Appendix 1. Since the Black-Scholes option pricing model already contains a variable of volatility, I will skip explicitly solving for the bank's loss in this paper.

4 Key findings and predictions

In this section, I will summarize the main findings from the model solutions, as well as relationship to the empirical literature discussed earlier in this paper.

1. Sellers with high LTV ratios set and receive higher prices for their properties. The bank also maximizes their claim when the homeowner has a higher LTV ratio, but that comes with higher potential losses associated with the mortgage.

2. When faced with a high interest rate, future homeowners will be more likely to put more equity towards their home.

3. Sellers who contribute more equity to the house value receive lower interest rates from the bank. Moreover, if the time to maturity is short, the interest rate becomes very low even with a relatively small equity contribution.

Now, I will discuss a few testable predictions and will suggest possible empirical tests of these predictions.

4.1 Testable predictions

To test some of the results derived in the previous sections and to guide further research on the questions posed in this paper, I propose two testable predictions below:

1. Owners who provide no equity when buying a home indeed receive the highest interest rates, regardless of what their other credit characteristics (such as annual salary, existing debt, previous record of home ownership) look like.

   H_0: The interest rate for all-debt homeowners is independent of other credit characteristics.

   H_a: The interest rate for all-debt homeowners has a statistically significant relationship with other credit characteristics.

   Interest rates on consumer debt depend on many different characteristics that a borrower has. As shown in the previous section, when a homeowner takes on mortgage debt, his equity input matters in determining how large the interest rate will be. However, typical credit applications also make inquiries about several other personal details – which may include annual salary, existing consumer debt, and whether the person owned a home in the past. All of these characteristics may be indicative of how trustworthy and financially solvent a borrower is and whether they will be able to repay their loan on time. One way to investigate the null hypothesis stated above is to use mortgage application data and check for relationships between different characteristics (besides the equity input) and the interest rate that homeowners received on their mortgage.

2. Banks advertise their mortgage loans in a way that encourages potential homeowners to put up their own money towards the home purchase, given that these potential homeowners are risk-averse.

   H_0: When given an upfront option of receiving a lower interest rate in exchange for putting a certain amount of equity towards the house value, homeowners will choose to put more equity compared to when they were not given such option.

H_a: When given an upfront option of receiving a lower interest rate in exchange for putting a certain amount of equity towards the house value, homeowners will not put more equity compared to when they were not given such option.

As seen in the previous section, putting up more equity towards the home purchase allows potential homeowners to get a lower interest rate on their mortgage. But it is unclear whether the banks actually encourage them to do that or whether they intentionally want their customers to borrow as large of a fraction of the total home value as possible. Given that homeowners with higher leverage tend to take on higher risks when they sell their home via a higher listing price, the bank might want to stimulate homeowners to provide more of equity by offering them a lower interest rate if they meet a certain equity threshold (different for each loan term). Those homeowners who were told about the opportunity upfront are expected to put up more equity compared to those who weren't.

5 Conclusion

To conclude my discussion, the model solution tends to be in line with the empirical facts stated in the literature review. I recognize that many modeling choices used are very restrictive and may not reflect all subtleties that exist in sellers' determination of listing prices. Moreover, by restricting the indebtedness measure to the LTV ratio, which is based purely on the mortgage value, I do not account for many other credit factors that may influence risk-shifting by house sellers. A more comprehensive credit model is thus needed to estimate all possible sources of financial solvency/insolvency and their effect on the decisions around property listing prices, as well as the effect on mortgage interest rates. Finally, further research on all unobservable characteristics of sellers (and not only the characteristics discussed in this paper) is necessary to have a more robust understanding of the real estate market sell-side.

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All graphs were modeled in Desmos.

6 Appendix

6.1 Proofs to the propositions

6.1.1 Proof to Proposition 1 and Corollary 1

Let $L' < L^* = 1, L_{HLTV} \geq L' \geq L_{LLTV}$. By a constraint set earlier, $0 \geq L \geq 1$. Set $\theta = 1$. Then for such L', we can calculate the homeowner's proceeds from house sale as

$$N = \theta[P_{sell} - L'P_m] \geq 0,$$

which converts to

$$N = P_{sell} - L'P_m \geq 0.$$

For L^*, the proceeds from house sale are equal to

$$N = P_{sell} - L * P_m = P_{sell} - P_m \geq 0.$$

Assume $P_{sell} - L'P_m > P_{sell} - P_m$. But since $P_{sell,LLTV} < P_{sell,HLTV}$, this cannot be true. Thus, it should be the case that $L^* = 1$.

6.1.2 Proof to Proposition 3 (Loss with simple European call option)

Proof to 3a. Let $L' > L^* = 0$. Set $\theta = 1$ and $P_m = V_h$. Then we can calculate the bank's proceeds from holding the homeowner's debt:

$$V_h - \theta[P_{sell} - D(1+r)^T] = V_h - P_{sell} + D'(1+r)^T = V_h - P_{sell} + V_h L'(1+r)^T = V_h(1 + L'(1+r)^T) - P_{sell}.$$

The bank's proceeds from holding L^* are

$$V_h - \theta[P_{sell} - D(1+r)^T] = V_h - P_{sell} + D * (1+r)^T = V_h - P_{sell} + V_h L * (1+r)^T = V_h - P_{sell}.$$

Thus, by $L' > L^*, V_h(1 + L'(1+r)^T) - P_{sell} > V_h - P_{sell}$.

Since $P_{sell} \leq P_0$ and $V_h \approx P_m P_0$, the inequality becomes

$$P_0(1 + L'(1+r)^T) - P_0 > P_0 - P_0,$$
$$L'(1+r)^T > 0.$$

Since $L' > 0, r \geq 0, T > 0$, the inequality holds. Hence proven.

Proof to 3b. Let $\theta' < \theta^* = 1$. Set $P_m = V_h$ and hold L constant. Then we can calculate the bank's proceeds from holding the homeowner's debt:

$$V_h - \theta'[P_{sell} - D(1+r)^T] = V_h - \theta' P_{sell} + \theta' V_h L(1+r)^T = V_h(1 + \theta' L(1+r)^T) - \theta' P_{sell}.$$

The bank's proceeds from holding the homeowner's debt when $\theta = \theta^*$ are

$$V_h - \theta[P_{sell} - D(1+r)^T] = V_h - P_{sell} + D(1+r)^T = V_h - P_{sell} + V_h L(1+r)^T = V_h(1 + L(1+r)^T) - P_{sell}.$$

Now assume that $V_h(1 + \theta' L(1+r)^T) - \theta' P_{sell} > V_h(1 + L(1+r)^T) - P_{sell}$.

Since $P_{sell} \leq P_0$ and $V_h \approx P_m \leq P_0$, the inequality becomes

$$(1 + \theta' L(1+r)^T) - \theta' > (1 + L(1+r)^T) - 1,$$

which we can convert to

$$\theta' L(1+r)^T - \theta' > L(1+r)^T - 1.$$

Dividing both sides by $L(1+r)^T - 1$, we get

$$\theta' > 1,$$

which is a contradiction, since $\theta' < \theta^* = 1$. Thus, $\theta' < 1$.

Hence proven.